

Grade 3 • Module 3

Multiplication and Division with Units of 0, 1, 6–9, and Multiples of 10

OVERVIEW

This 27-day module builds directly on students' work with multiplication and division in Module 1. At this point, Module 1 instruction coupled with fluency practice in Module 2 has students well on their way to meeting the Grade 3 fluency expectation for multiplying and dividing within 100 (**3.4E**). Module 3 extends the study of factors from 2, 3, 4, 5, and 10 to include all units from 0 to 10, as well as multiples of 10 within 100. Additionally students will multiply two-digit numbers by one-digit numbers, and record their work in both the partial product algorithm and the standard algorithm. Similar to the organization of Module 1, the introduction of new factors in Module 3 spreads across topics. This allows students to build fluency with facts involving a particular unit before moving on. The factors are sequenced to facilitate systematic instruction with increasingly sophisticated strategies and patterns.

Topic A begins by building on students' understanding of multiplication by exploring multiplication as a means to compare one quantity to another. (**3.5C**) Students will work from simple to complex, starting with strips of paper, transitioning to strip diagrams and then using number pairs in a table. While they do this work, students are enhancing their fluency of basic multiplication and division facts. (**3.4F**) The topic ends with a focus on multiplicative comparison word problems.

Topic B begins by revisiting the commutative property. Students study familiar facts from Module 1 to identify known facts using units of 6, 7, 8, and 9 (**3.4E, 3.4K**). They realize that they already know more than half of their facts by recognizing, for example, that if they know 2×8 , they also know 8×2 through commutativity. This begins a study of arithmetic patterns that becomes an increasingly prominent theme in the module (**3.4I**). The subsequent lesson carries this study a step further; students apply the commutative property to relate 5×8 and 8×5 and then add one more group of 8 to solve 6×8 and, by extension, 8×6 . The final lesson in this topic builds fluency with familiar multiplication and division facts, preparing students for the work ahead by introducing the use of a blank box to represent the unknown in various positions (**3.4K, 3.5D**).

Topic C introduces units of 6 and 7, factors that are well suited to Level 2 skip-counting strategies and to the Level 3 distributive property strategy, already familiar from Module 1. Students learn to compose up to and then over the next ten. For example, to solve a fact using units of 7, they might count 7, 14, and then mentally add $14 + 6 + 1$ to make 21. This strategy previews the associative property using addition and illuminates patterns as students apply count-bys to solve problems. Topic B's final lesson emphasizes word problems, providing opportunities to analyze and model. Students apply the skill of using a blank box to represent the unknown in various positions within multiplication and division problems (**3.4E, 3.4K, 3.5D**).



Topic D anticipates the formal introduction of the associative property with a lesson focused on making use of structure to problem solve. Students learn the conventional order for performing operations when parentheses are and are not present in an equation (**3.4K, 3.5A, 3.5B**). With this student knowledge in place, the associative property emerges in the next lessons as a strategy to multiply using units up to 8 (**3.4K**).

Units of 6 and 8 are particularly useful for presenting this Level 3 strategy. Rewriting 6 as 2×3 or 8 as 2×4 makes shifts in grouping readily apparent (see example on next page) and also utilizes the familiar factors 2, 3, and 4 as students learn the new material. The following strategy may be used to solve a problem like 8×5 :

$$8 \times 5 = (4 \times 2) \times 5$$

$$8 \times 5 = 4 \times (2 \times 5)$$

$$8 \times 5 = 4 \times 10$$

Topic E introduces units of 9, with students exploring a variety of arithmetic patterns that become engaging strategies for quickly learning facts with automaticity (**3.4E, 3.4K, 3.5E**). Nines are placed late in the module so that students have enough experience with multiplication and division to recognize, analyze, and apply the commutative and distributive properties to solve expressions including 9 as a factor. The topic ends with interpreting the unknown factor to solve multiplication and division problems (**3.4E, 3.4K, 3.5D**).

In Topic F, students begin by working with facts using units of 0 and 1. From a procedural standpoint, these are simple facts that require little time for students to master; however, understanding the concept of nothing (zero) is more complex, particularly as it relates to division. This unique combination of simple and complex explains the late introduction of 0 and 1 in the sequence of factors. Students study the results of multiplying and dividing with units of 0 and 1 to identify relationships and patterns (**3.4E, 3.4I, 3.5E**). The topic closes with a lesson devoted to two-step problems involving all four operations (**3.4K, 3.5A, 3.5B**). In this lesson, students work with equations involving unknown quantities and apply the rounding skills learned in Module 2 to make estimations that help them assess the reasonableness of their solutions (**3.4K, 3.5A, 3.5B**).

In Topic G, students multiply by multiples of 10 (**3.4F, 3.4G**). To solve a fact like 2×30 , they first model the basic fact 2×3 on the place value chart. Place value understanding helps them to notice that the product shifts one place value to the left when multiplied by 10: 2×3 tens can be found by simply locating the same basic fact in the tens column.

hundreds	tens	ones
		○○○
		○○○
		$2 \times 3 = 6$

hundreds	tens	ones
	○○○	
	○○○	
	$2 \times 3 \text{ tens} = 6 \text{ tens}$	
	$6 \text{ tens} = 60$	

In the subsequent lessons, students build on this understanding to multiply a two-digit number by a one-digit number (**3.4G**). Lessons proceed from concrete to abstract. In Lesson 20, students use place value disks and record work using a partial products algorithm.

In Lesson 21, students continue their work of multiplying a two-digit number by a one-digit number by representing the disks on a labeled place value chart, and still recording their work with a partial products algorithm. The standard algorithm is introduced in Lesson 22, and practiced with two-step word problems in Lesson 23 (3.4A). Embedded in these lessons, students continue to practice basic facts in order to gain automaticity. For example, when multiplying 6 times 4 tens, students are practicing the basic fact of 6×4 (3.4F).

53×3

$$\begin{array}{r} 53 \\ \times 3 \\ \hline 9 \leftarrow 3 \times 3 \text{ ones} \\ + 150 \leftarrow 3 \times 5 \text{ tens} \\ \hline 159 \end{array}$$

$$3 \begin{array}{|c|c|} \hline 150 & 9 \\ \hline \end{array}$$

Focus Grade Level Standards

Number and Operations

The student applies mathematical process standards to develop and use strategies and methods for whole number computations in order to solve problems with efficiency and accuracy. The student is expected to:

- 3.4E** represent multiplication facts by using a variety of approaches such as repeated addition, equal-sized groups, arrays, area models, equal jumps on a number line, and skip counting.
- 3.4F** recall facts to multiply up to 10 by 10 with automaticity and recall the corresponding division fact.
- 3.4G** use strategies and algorithms, including the standard algorithm, to multiply a two-digit number by a one-digit number. Strategies may include mental math, partial products, and the commutative, associative, and distributive properties.
- 3.4I** determine if a number is even or odd using divisibility rules.
- 3.4K** solve one-step and two-step problems involving multiplication and division within 100 using strategies based on objects; pictorial models, including arrays, area models, and equal groups; properties of operations; or recall of facts.

Algebraic Reasoning

The student applies mathematical process standards to analyze and create patterns and relationships. The student is expected to:

- 3.5A** represent one- and two-step problems involving addition and subtraction of whole numbers to 1,000 using pictorial models, number lines, and equations.
- 3.5B** represent and solve one- and two-step multiplication and division problems within 100 using arrays, strip diagrams, and equations.
- 3.5C** describe a multiplication expression as a comparison such as 3×24 represents 3 times as much as 24;
- 3.5D** determine the unknown whole number in a multiplication or division equation. relating three whole numbers when the unknown is either a missing factor or product.
- 3.5E** represent real-world relationships using number pairs in a table and verbal descriptions.

Foundational Standards

Number and Operations

The student applies mathematical process standards to understand how to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system related to place value. The student is expected to:

- 2.2C** generate a number that is greater than or less than a given whole number up to 1,200

Number and Operations

The student applies mathematical process standards to develop and use strategies and methods for whole number computations in order to solve problems with efficiency and accuracy. The student is expected to:

- 3.4D** determine the total number of objects when equally-sized groups of objects are combined or arranged in arrays up to 10 by 10.
- 3.4E** represent multiplication facts by using a variety of approaches such as repeated addition, equal-sized groups, arrays, area models, equal jumps on a number line, and skip counting.
- 3.4H** determine the number of objects in each group when a set of objects is partitioned into equal shares or a set of objects is shared equally.
- 3.4J** determine a quotient using the relationship between multiplication and division.

Number and Operations

The student applies mathematical process standards to connect repeated addition and subtraction to multiplication and division situations that involve equal groupings and shares. The student is expected to:

- 2.6A** model, create, describe contextual multiplication situations in which equivalent sets of concrete objects are joined.

Algebraic Reasoning

The student applies mathematical process standards to analyze and create patterns and relationships. The student is expected to:

- 3.5D** determine the unknown whole number in a multiplication or division equation relating three whole numbers when the unknown is either a missing factor or product.

Algebraic Reasoning

The student applies mathematical process standards to identify and apply number patterns within properties of numbers and operations in order to describe relationships. The student is expected to:

- 2.7A** determine whether a number up to 40 is even or odd using pairings of objects to represent the number.

Focus Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

- MPS(D)** communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
- MPS(E)** create and use representations to organize, record, and communicate mathematical ideas;
- MPS(F)** analyze mathematical relationships to connect and communicate mathematical ideas.

Overview of Module Topics and Lesson Objectives

TEKS	ELPS	Topics and Objectives	Days
3.5C 3.5E 3.4F	1.C 1.E 2.G 3.F 4.G 5.G	A Multiplication as Comparison Lessons 1–2: Use multiplication to compare. Lesson 3: Use tables to record multiplicative relationships. Lesson 4: Solving multiplicative comparison word problems.	4
3.4E 3.4K 3.5D 3.5E 3.4D 3.4H	1.C 2.E 2.I 3.D 3.H 4.B 5.G	B The Properties of Multiplication and Division Lesson 5: Study commutativity to find known facts of 6, 7, 8, and 9. Lesson 6: Apply the distributive and commutative properties to relate multiplication facts $5 \times n + n$ to $6 \times n$ and $n \times 6$ where n is the size of the unit. Lesson 7: Multiply and divide with familiar facts using a box to represent the unknown.	3



TEKS	ELPS	Topics and Objectives	Days
3.4E 3.4K 3.5D 3.4D 3.4H	1.F 2.C 2.D 2.E 2.I 3.E 3.F 4.B	C Multiplication and Division Using Units of 6 and 7 Lesson 8: Count by units of 6 to multiply and divide using number bonds to decompose. Lesson 9: Count by units of 7 to multiply and divide using number bonds to decompose. Lesson 10: Interpret the unknown in multiplication and division to model and solve problems using units of 6 and 7.	3
3.4E 3.4K 3.5D 3.4D 3.4H 3.5A 3.5B	2.H 2.I 3.E 3.H 4.G 5.G	D Multiplication and Division Using Units up to 8 Lesson 11: Understand the function of parentheses and apply to solving problems. Lesson 12: Model the associative property as a strategy to multiply.	2
		Mid-Module Assessment: Topics A–D (assessment ½ day, return ½ day, remediation or further applications 1 day)	2
3.4E 3.4K 3.5D 3.5E 3.4D 3.4H	2.E 2.I 3.E 3.H 4.B 4.G 5.G	E Multiplication and Division Using Units of 9 Lesson 13: Apply the distributive property and the fact $9 = 10 - 1$ as a strategy to multiply. Lesson 14: Interpret the unknown in multiplication and division to model and solve problems.	2
3.4E 3.4I 3.4K 3.5A 3.5B 3.5E 3.4D 3.4H 3.5D	1.C 2.E 2.I 3.E 3.G 3.H 4.F 4.G 5.G	F Analysis of Patterns and Problem Solving Including Units of 0 and 1 Lesson 15: Reason about and explain arithmetic patterns using units of 0 and 1 as they relate to multiplication and division. Lesson 16: Identify patterns in multiplication and division facts using the multiplication table. Lesson 17: Solve two-step word problems involving all four operations and assess the reasonableness of solutions.	3



TEKS	ELPS	Topics and Objectives	Days
3.4F 3.4G 3.4K 3.5A 3.5B 3.5E 3.4D 3.4E	1.C 2.E 2.I 2.G 3.G 3.H 4.C 4.G 5.G	G Multiplication of Single-Digit Factors and Two-Digit Factors Lesson 18: Multiply by multiples of 10 using the place value chart. Lesson 19: Use place value strategies and the associative property $n \times (m \times 10) = (n \times m) \times 10$ (where n and m are less than 10) to multiply by multiples of 10. Lesson 20: Use concrete models to represent two-digit by one-digit multiplication. Lesson 21: Draw models to represent two-digit by one-digit multiplication. Lesson 22: Multiply two-digit numbers by one-digit numbers using the standard algorithm. Lesson 23: Solve two-step word problems involving multiplying single-digit factors by multiples of 10 and two-digit factors.	6
		End-of-Module Assessment: Topics A–G (assessment ½ day, return ½ day, remediation or further application 1 day)	2
Total Number of Instructional Days			27

Terminology

New or Recently Introduced Terms

Input-output table (a two column or two row table with inputs in the first column, or row, and their corresponding outputs in the second column, or row)

- Multiple (specifically with reference to naming multiples of 9 and 10, e.g., 20, 30, 40, etc.)
- Partial Product (e.g., $24 \times 6 = (20 \times 6) + (4 \times 6) = 120 + 24$)
- Product (the quantity resulting from multiplying two or more numbers together)

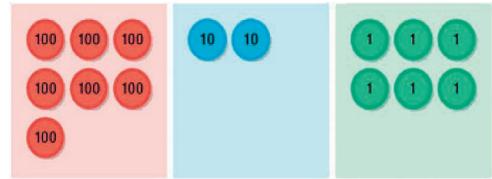
Familiar Terms and Symbols¹

- Array (a set of numbers or objects that follow a specific pattern)
- Commutative property (e.g., $2 \times 3 = 3 \times 2$)
- Distribute (with reference to the distributive property; e.g., in $12 \times 3 = (10 \times 3) + (2 \times 3)$, the 3 is the multiplier for each part of the decomposition)
- Divide, division (partitioning a total into equal groups to show how many equal groups add up to a specific number, e.g., $15 \div 5 = 3$)
- Equal groups (with reference to multiplication and division; one factor is the number of objects in a group, and the other is a multiplier that indicates the number of groups)
- Equation (a statement that two expressions are equal, e.g., $3 \times 4 = 12$)
- Even number (a whole number whose last digit is 0, 2, 4, 6, or 8)
- Expression (a number, or any combination of sums, differences, products, or divisions of numbers that evaluates to a number, e.g., 8×3 , $15 \div 3$)
- Factors (numbers that are multiplied to obtain a product)
- Multiply, multiplication (an operation showing how many times a number is added to itself, e.g., $5 \times 3 = 15$)
- Number bond (model used to show part–part–whole relationships)
- Number sentence (an equation or inequality for which both expressions are numerical and can be evaluated to a single number, e.g., $21 > 7 \times 2$, $5 \div 5 = 1$)
- Odd number (a number that is not even)
- Ones, twos, threes, etc. (units of one, two, or three)
- Parentheses (the symbols () used around a fact or numbers within an equation, expression, or number sentence)
- Quotient (the answer when one number is divided by another)
- Row, column (in reference to rectangular arrays)
- Strip diagram (a method for modeling problems)
- Unit (one segment of a partitioned strip diagram)
- Unknown (the *missing* factor or quantity in multiplication or division)
- Value (how much)

¹These are terms and symbols students have used or seen previously.

Suggested Tools and Representations

- Array
- Number bond (model used to show part–part–whole relationships)
- Place value disks (pictured at right)
- Strip diagram (a method for modeling problems)



Scaffold

The scaffolds integrated into *A Story of Units*® give alternatives for how students access information as well as express and demonstrate their learning. Strategically placed margin notes are provided within each lesson elaborating on the use of specific scaffolds at applicable times. They address many needs presented by English language learners, students with disabilities, students performing above grade level, and students performing below grade level. Many of the suggestions are organized by Universal Design for Learning (UDL) principles and are applicable to more than one population.

Assessment Summary

Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic D	Constructed response with rubric	3.4E 3.4K 3.5C 3.5D 3.5E
End-of-Module Assessment Task	After Topic G	Constructed response and timed fluency with rubric	3.4F 3.4K 3.5A 3.5B 3.5D 3.5E