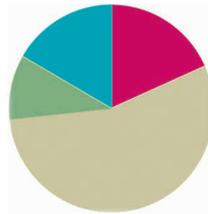


Lesson 5

Objective: Use a circular protractor to understand a 1-degree angle as $\frac{1}{360}$ of any circle. Explore benchmark angles using the protractor.

Suggested Lesson Structure

■ Fluency Practice	(11 minutes)
■ Application Problem	(6 minutes)
■ Concept Development	(33 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (11 minutes)

- Divide Using the Standard Algorithm **4.4E, 4.4F** (3 minutes)
- Identify Two-Dimensional Figures **4.6A** (4 minutes)
- Physiometry **4.6A** (4 minutes)

Divide Using the Standard Algorithm (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Grade 4 Module 3 Lesson 16 content.

T: (Write $48 \div 2$.) On your personal white boards, solve the division problem using the vertical method.

Continue with the following possible sequence: $49 \div 2$, $69 \div 3$, $65 \div 3$, $55 \div 5$, $58 \div 5$, $88 \div 4$, and $86 \div 4$.

Identify Two-Dimensional Figures (4 minutes)

Materials: (S) Personal white board, straightedge

Note: This fluency activity reviews terms learned in Lessons 1–4.

T: (Project \overleftrightarrow{AB} . Point to A .) Say the term for what I'm pointing to.

S: Point A .

T: (Point to B .) Say the term.

S: Point B .

T: (Point to \overleftrightarrow{AB} .) Say the term.

S: Line AB .

T: Use your straightedge to draw \overleftrightarrow{CD} on your personal white boards.

S: (Draw a line with points C and D on the line.)

Continue with the following possible sequence: \overleftrightarrow{EF} , \overleftrightarrow{GH} , and obtuse $\angle IKJ$, acute $\angle LNM$, and right $\angle OQP$.

T: What's the relationship between \overleftrightarrow{QO} and \overleftrightarrow{QP} ?

S: The line segments are perpendicular.

T: Draw \overleftrightarrow{RS} .

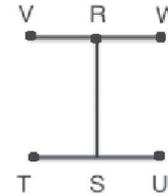
S: (Draw \overleftrightarrow{RS} .)

T: Draw \overleftrightarrow{TU} that is perpendicular to \overleftrightarrow{RS} .

S: (Draw \overleftrightarrow{TU} .)

T: Draw \overleftrightarrow{VW} that is perpendicular to \overleftrightarrow{RS} and parallel to \overleftrightarrow{TU} .

S: (Draw \overleftrightarrow{VW} .)



Physiometry (4 minutes)

Note: Kinesthetic memory is strong memory. This fluency activity reviews terms from Lessons 1–4.

T: Stand up.

S: (Stand up.)

T: Show me a point.

S: (Clench one hand in a fist, and extend arm forward.)

T: Show me a line.

S: (Extend arms straight so that they are parallel with the floor. Open both hands.)

T: Show me a ray.

S: (Extend arms straight so that they are parallel with the floor. Clench one hand in a fist, and leave the other hand open.)

T: Show me a ray pointing in the other direction.

S: (Open clenched hand, and clench open hand.)

T: Show me a line segment.

S: (Extend arms straight so that they are parallel with the floor. Clench both hands into fists.)

T: Show me a right angle.

S: (Stretch one arm up directly at the ceiling. Stretch another arm directly toward a wall, parallel to the floor.)

T: Show me a different right angle.

S: (Stretch the arm pointing toward a wall directly up toward the ceiling. Move the arm pointing toward the ceiling so that it points directly toward the opposite wall.)

T: Show me an obtuse angle.

S: (Make an obtuse angle with arms.)

T: Show me an acute angle.

S: (Make an acute angle with arms.)

Continue with the following possible sequence: point, right angle, line segment, acute angle, line, right angle, and obtuse angle.

T: (Stretch one arm up directly at the ceiling. Stretch another arm directly toward a wall, parallel to the floor.) What type of angle am I making?

S: Right angle.

T: What is the relationship of the lines formed by my arms?

S: Perpendicular lines.

T: (Point to the classroom's back wall.) Point to the walls that run perpendicular to the wall I'm pointing to.

S: (Point to the side walls.)

T: (Point to the front wall.)

S: (Point to the side walls.)

Continue pointing to one side wall, the back wall, the other side wall, and the front wall.

T: (Point to the back wall.) Point to the wall that runs parallel to the wall I'm pointing to.

S: (Point to the front wall.)

Continue pointing to one side wall, the front wall, and the other side wall.

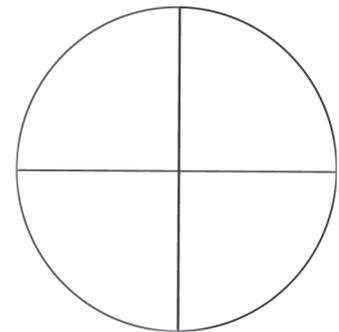
Application Problem (6 minutes)

Materials: (S) 1 paper circle from the Concept Development

Place right angle templates on top of the circle to determine how many right angles can fit around the center point of the circle. If necessary, team up with other students to share templates. (Overlaps are not allowed.)

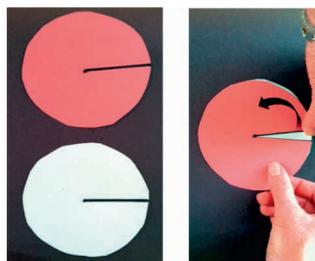
How many right angles can fit?

Note: This Application Problem bridges concepts from Lesson 2. Students use the right angle templates that they made in class to build understanding as they measure right angles around the center point of a circle.



Concept Development (33 minutes)

Materials: (T/S) 2 paper circles (5-inch diameter—one red, one white) with a radius cut into each one, delineated by a strong, straight black segment, circular protractor (Template) printed on paper or transparency



Directions for Constructing a Paper Protractor:

1. Label and cut a radius into one red and one white paper circle.
2. Stack the red circle on top of the white circle with the radii aligned and parallel to the desk's edge.
3. Pinch the corner of the white circle directly below the slit (as shown on the previous page).
4. To illustrate an angle, turn the segment given by the edge of the white region counterclockwise.

Problem 1: Reason about the number of turns necessary to make a full turn with different fractions of a full turn.

T: What do you see as you turn this segment to the left?

S: The white part is getting larger. The red part is getting smaller.

T: Do you see an angle? What is the vertex of this angle? Where are the rays?

S: Yes. The center of the circle is the vertex of the angle. The rays are where the red and white regions meet.

T: Let's agree that the white region is the interior of the angle we are focusing on.

T: (Demonstrate a quarter-turn.) Now, show a quarter-turn of the segment to the left. (Expect some confusion.)

S: (Show.)

T: Make another quarter-turn of the segment to the left. What fraction of the circular region is white now?

S: One-half. → Two-fourths.

T: If we cut out the angle represented by the white region, what fraction of the circle would we cut out?

S: One half of the circle. → Two-fourths of the circle.

Continue the same process until the 360° turn is complete.

T: (List the following fractions on the board.)

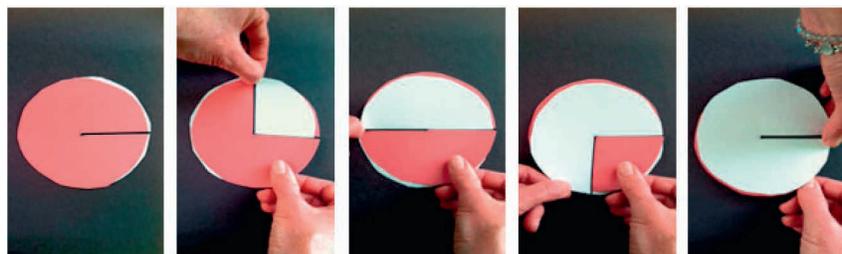
$$\frac{1}{4} \quad \frac{2}{4} \quad \frac{3}{4} \quad \frac{4}{4}$$

T: (Point to each fraction while speaking, pausing as students manipulate the turns.) Show $\frac{1}{4}$ turn, $\frac{2}{4}$ turn, now a $\frac{3}{4}$ turn, a $\frac{4}{4}$ turn. Is the angle getting larger or smaller?

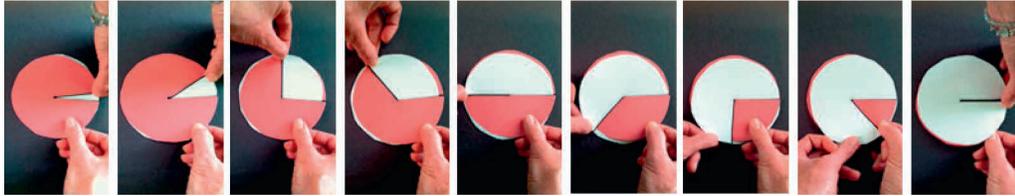
S: Larger!

T: How many fourth-turns did it take to make one full turn?

S: Four.



T: Now, I want to break up a turn into eight equal parts. Count eighths with me.



T: Will one eighth-turn, or one-eighth cut from the circle, be less than or greater than one-fourth turn or one-fourth cut from the circle?

S: Less than

T: One fourth turn is the same as two eighth-turns (point to the listed fractions). Show me what you think one-eighth cut from the circle might look like.

Repeat the same process of pointing to each one-eighth cut from the circle, in order, as the students open the angle.

T: Did it take more fourth-turns - fourths cut from the circle - or eighth-turns - eighths cut from the circle to go around the whole circle?

S: Eighths.

T: How many eighth-turns did it take to move all the way around? Or to ask another way, how many eighths could we cut from a whole circle?

S: Eight!

T: How many $\frac{1}{100}$ turns would it take to make a whole turn? In other words, how many hundredths could we cut from a whole circle?

S: 100.

T: Would a $\frac{1}{360}$ cut from a circle be smaller or larger than a $\frac{1}{100}$ cut from a circle? Would a $\frac{1}{360}$ turn be smaller or larger than a $\frac{1}{100}$ turn?

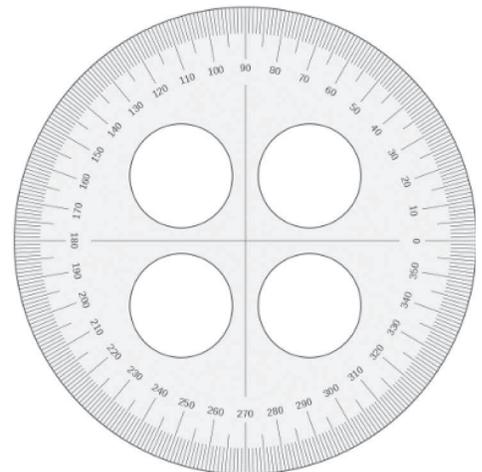
S: Smaller.

T: We have a special name for a $\frac{1}{360}$ of a whole turn, or $\frac{1}{360}$ cut from a circle. It is called a **degree**! How many degrees are in one whole circle or a whole turn?

S: 360° .

T: Yes!

T: Here is a tool that has been partitioned and marked off to show 360° . It is called a **protractor**. The **degree measure of an angle** is measured by a protractor. Take a moment to analyze the protractor with your partner. What do you notice?



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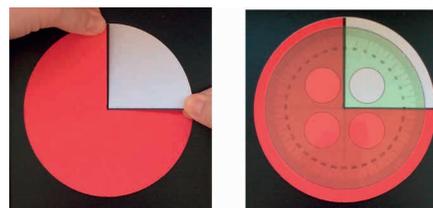
- S: It is shaped like a circle. → It is counting by tens, starting at zero and going to the left. → Between each ten are tick marks showing the ones. → It has 360° . → I see four right angles! → The four right angles have the numbers 0, 90, 180, and 270.
- T: Run your finger across your protractor from zero to the center point where the bold perpendicular lines cross. Let's call that the zero line, or base line, of our protractor because it will be the starting point from where we measure angles.

Problem 2: Use a circular protractor to determine that a quarter-turn or a right angle measures 90° , a half-turn measures 180° , a three quarter-turn measures 270° , and a full rotation measures 360° .

- T: Show me a quarter-turn with your circles. Keep the base segment of your angle parallel to your desk.
- T: Put the zero line, or base line, on top of the bottom segment of your angle. Align the center point of the protractor with the vertex of the angle to the best of your ability.
- T: Adjust the circle's angle to match your right angle template. (Pause.) Remove the template and place the protractor to measure that angle. What do you notice?
- S: The quarter-turn matches the bold lines of the protractor. → It's 90° . → One-fourth cut out of 360° is 90° . → A right angle measures 90° .
- T: Do a half-turn. How many degrees are in your angle? How many degrees are in one-half cut from a circle?
- S: 180° .
- T: Turn another quarter- or fourth-turn. How many degrees are in three one-fourth cut outs of a circle?
- S: 270° .
- T: And one last quarter- or fourth-turn?
- S: 360° . → 0° .
- T: What does your angle look like right now?
- S: It's all white.
- T: A zero-degree angle is when we have not turned at all.

We have made one full turn of 360° . There are 360° in a full turn. A circle with nothing cut out is 360° .

- T: How many 90° angles, or right angles, are there in a whole circle?
- S: Four right angles.
- T: How do you know?
- S: Because we made four quarter-turns, which is the same as four one-fourth cut outs of a circle, and each one was 90° . → It is easy to see them because of the bold perpendicular lines on the protractor.



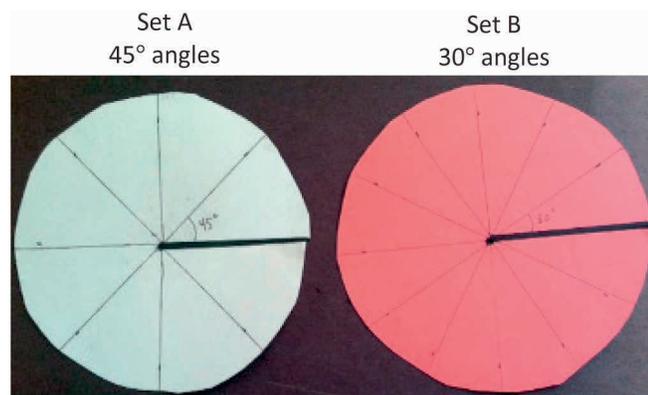
NOTES ON PROTRACTORS:

Circular protractors come in many different sizes and formats. Several include not one but two numbered scales. Since the lesson's objective is to understand a one-degree angle, it is recommended to use the template, which only has one set of angle measures moving counter-clockwise counting by tens. Its simplicity may help ground students' initial understanding so that when they encounter a *regular* protractor in the next lesson, they will have internalized the meaning of a degree and the measures of acute and obtuse angles.

- T: Using your white circle, position your protractor with the zero or base line on top of the black segment, matching up the center point of the circle with the center point of the protractor.
- T: Estimate to make a point at 90° . Draw a line segment from the center point to that point. What have you drawn?
- S: A right angle. \rightarrow A 90° angle. \rightarrow Perpendicular lines.
- T: Now, make a point at 45° . Draw a line segment from the center point to the point you just made. What have you made?
- S: A 45° angle.
- T: Yes! What do you notice?
- S: The 45° angle is half as big as the 90° angle.
 \rightarrow Two 45° angles are the same as one 90° angle.
 $\rightarrow 2 \times 45 = 90$.

Problem 3: Measure and draw benchmark angles with the protractor.

- T: Now, let's work to measure and draw benchmark angles using your circles and protractors.
- T: We have already started Set A using your white circle. Continue turning your circle, aligning the zero or base line with each last segment drawn. Be sure to keep your protractor's center point on the center point of the circle. Draw new 45° angles until you have made a whole turn. Let me demonstrate. (Demonstrate silently.)
- T: Draw Set B on your red circle just as you did for Set A, but this time, draw 30° angles. This full turn will be made of 30° angles. Draw 30° angles until you have made a whole turn.



- T: Place the center point of the protractor on the shared endpoints of the segments on your white circle. Align the zero line with the black segment. What are the measurements of the angles you have drawn?



**NOTES ON
MULTIPLE MEANS
OF ACTION AND
EXPRESSION:**

Ease the task of drawing benchmark angles for students with the following suggestions:

- Provide larger paper circles of thicker cardstock that may be easier to manipulate.
- Provide circles with pre-drawn markings.
- Provide the complete list of benchmark angles, liberating students to focus on reading and drawing angles with a protractor.

S: 0° , 45° , 90° , 135° , 180° , etc.

T: Trace each angle separately with your finger, moving from the smallest to largest angles.

If you cut along each line, how many cut outs of a circle would you have?

S: 8

T: So each one-eighth cut out of a circle measures how many degrees?

S: 45° .

Repeat the process with the sequence of 30° angles.

T: All of these are benchmark angles. Let's use our Problem Set to further explore them.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set. Some students might be allowed to complete the drawing of the benchmark angles, while others start on the Problem Set. Take 10 minutes for the Problem Set, as always, with the understanding that the variation in work completed may differ considerably.

Student Debrief (10 minutes)

Lesson Objective: Use a circular protractor to understand a 1-degree angle as $\frac{1}{360}$ of any circle. Explore benchmark angles using the protractor.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Name Jack Date _____

1. Make a list of the measures of the benchmark angles you drew starting with Set A. Round each angle measure to the nearest 5° . Both sets have been started for you.

a. Set A: 45° , 90° , 135° , 180° , 225° , 270° , 315° , 360°

b. Set B: 30° , 60° , 90° , 120° , 150° , 180° , 210° , 240° , 270° , 300° , 330° , 360°

2. Circle any angle measures that appear on both lists. What do you notice about them?
They are all quarter turns. They are all right angles.

3. List the angle measures from Problem 1 that are acute. Trace each angle with your finger as you say its measurement.
 30° , 45° , 60°

4. List the angle measures from Problem 1 that are obtuse. Trace each angle with your finger as you say its measurement.
 120° , 135° , 150°

Any combination of the questions below may be used to lead the discussion.

- When you listed the benchmark angles, did you notice any numerical patterns?
- You listed some measures of acute and obtuse angles. What would be some measurements of other acute angles? Obtuse angles?
- A complete circle is 360° . What could you do to find the **degree measure of an angle** that is one-tenth cut out of circle?
- How did you respond to the final question?
- If you were to draw a strip diagram to represent one whole circle and the benchmark angles of Set A, what would you do? Set B?
- Shade in the region of a 45° angle on your white circle. What fraction of a whole circle is that? Do the same for your 30° angle. What if you shaded in a region defined by a 120° angle on your red circle? What fraction of the whole circle is that?
- Use your **protractor** to explain to your partner what a **degree** is.
- Use your protractor to trace some benchmark angles.
- If you didn't have a protractor, how could you construct one?

5. We found out today that 1° is $\frac{1}{360}$ of a circle. It is 1 out of 360° . That means a 2° angle is $\frac{2}{360}$ of a circle. What fraction of a circle is each of the benchmark angles you listed in Problem 1?

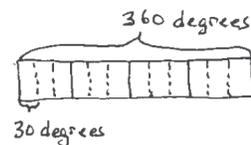
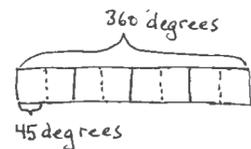
Set A: $\frac{45}{360}, \frac{90}{360}, \frac{180}{360}, \frac{225}{360}, \frac{270}{360}, \frac{315}{360}, \frac{360}{360}$

Set B: $\frac{30}{360}, \frac{60}{360}, \frac{90}{360}, \frac{120}{360}, \frac{150}{360}, \frac{180}{360}, \frac{210}{360}, \frac{240}{360}, \frac{270}{360}, \frac{300}{360}, \frac{330}{360}, \frac{360}{360}$

6. How many 45° angles does it take to make a full turn?
It takes eight 45° angles to make a full turn.

7. How many 30° angles does it take to make a full turn?
It takes twelve 30° angles to make a full turn.

8. If you didn't have a protractor, how could you reconstruct a quarter of it from 0° to 90° ?
*You could use two 45° angles or three 30° angles put together.
 You could make a right angle template.*



Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

5. We found out today that 1° is $\frac{1}{360}$ of a circle. It is 1 out of 360° . That means a 2° angle is $\frac{2}{360}$ of a circle. What fraction of a circle is each of the benchmark angles you listed in Problem 1?
6. How many 45° angles does it take to make a full turn?
7. How many 30° angles does it take to make a full turn?
8. If you didn't have a protractor, how could you reconstruct a quarter of it from 0° to 90° ?



Name _____

Date _____

1. How many right angles make a circle, or a full turn?

2. What is the measurement of a right angle?

3. What fraction of a circle, or a full turn is 1° ?

4. Name at least four benchmark angle measurements.



Lesson 5: Use a circular protractor to understand a 1-degree angle as $\frac{1}{360}$ of any circle. Explore benchmark angles using the protractor.

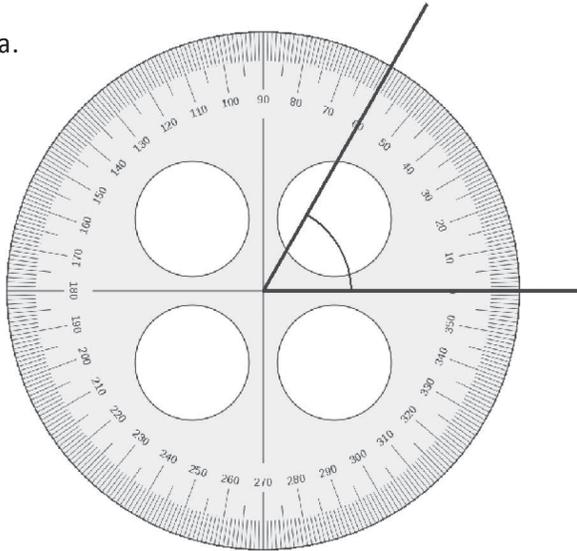


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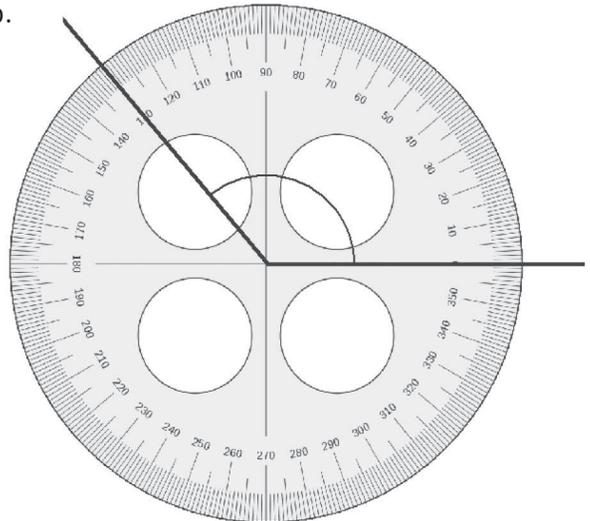
Date _____

1. Identify the measures of the following angles.

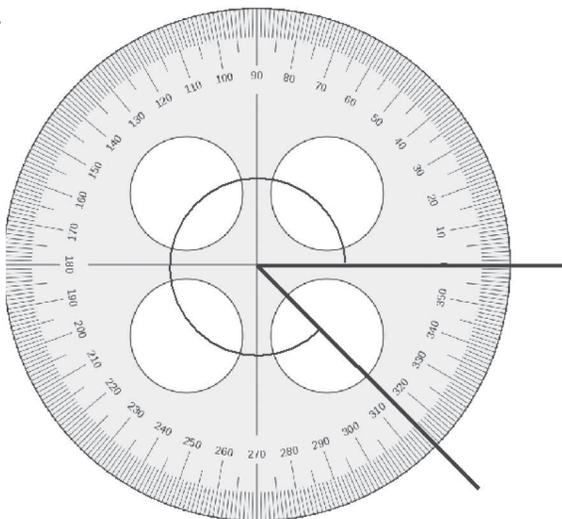
a.



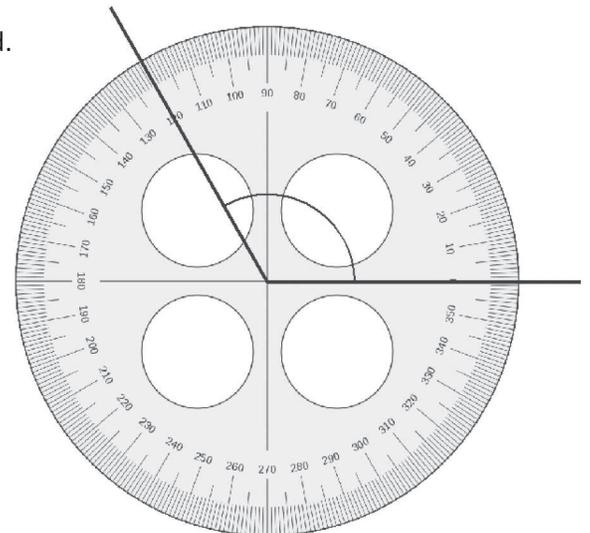
b.



c.



d.

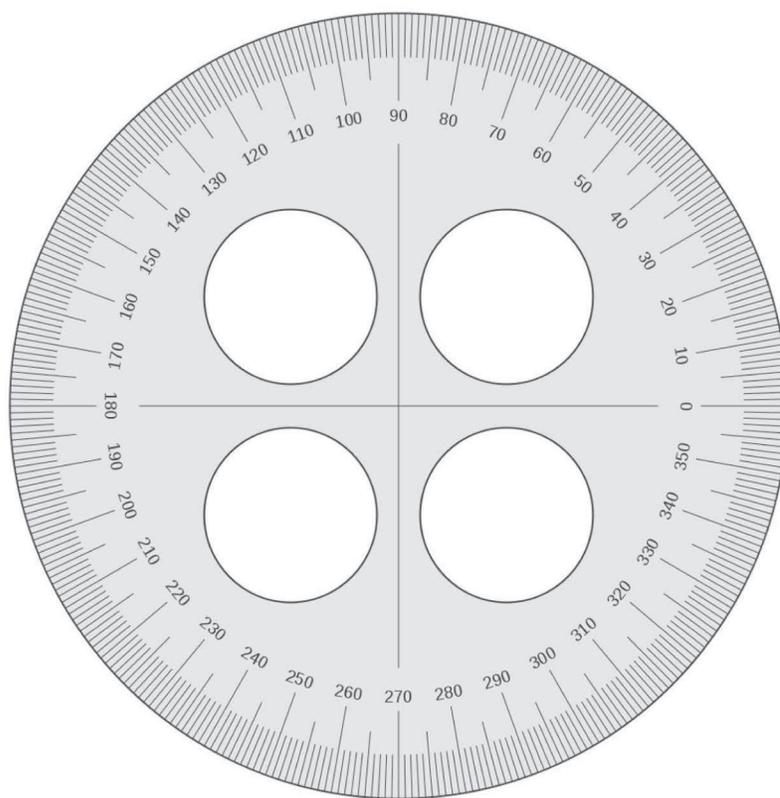


2. If you didn't have a protractor, how could you construct one? Use words, pictures, or numbers to explain in the space below.



Lesson 5: Use a circular protractor to understand a 1-degree angle as $\frac{1}{360}$ of any circle. Explore benchmark angles using the protractor.





circular protractor