

Grade 4 • Module 4

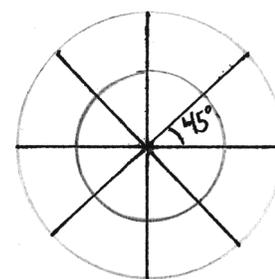
Angle Measure and Plane Figures

OVERVIEW

This 20-day module introduces points, lines, line segments, rays, and angles, as well as the relationships between them. Students construct, recognize, and define these geometric objects before using their new knowledge and understanding to classify figures and solve problems. With angle measure playing a key role in the work throughout the module, students learn how to create and measure angles, as well as how to create and solve equations to find unknown angle measures. In these problems, where the unknown angle is represented by a letter, students explore both measuring the unknown angle with a protractor and reasoning through the solving of an equation. This connection between the measurement tool and the numerical work lays an important foundation for success with middle-school geometry and algebra. Through decomposition and composition activities, as well as an exploration of symmetry, students recognize specific attributes present in two-dimensional figures. They further develop their understanding of these attributes as they classify two-dimensional figures.

Topic A begins with students drawing points, lines, line segments, and rays, as well as identifying these in various contexts and within familiar figures. Students recognize that two rays sharing a common endpoint form an angle (**4.7A, 4.7B**). They create right angles through a paper-folding activity, identify right angles in their environment, and see that one angle can be greater (obtuse) or less (acute) than a right angle. Next, students use their understanding of angles to explore relationships between pairs of lines as they define, draw, and recognize intersecting, perpendicular, and parallel lines (**4.6A**).

In Topic B, students explore the definition of degree measure, beginning with a circular protractor. By dividing the circumference of a circle into 360 equal parts, they recognize one part as representing 1 degree (**4.7A, 4.7B**). Through exploration, students realize that, although the size of a circle may change, an angle spans an arc, representing a constant fraction of the circumference. By carefully distinguishing the attribute of degree measure from that of length measure, the common misconception that degrees are a measure of length is avoided. Armed with their understanding of the degree as a unit of measure, students use various types of protractors to measure angles to the nearest degree and to sketch angles of a given measure (**4.7C, 4.7D**). The idea that an angle measures the amount of *turning* in a particular direction and the notion that the measure of an angle represents a fraction of a circle that is *cut out* are explored as students recognize familiar angles in varied contexts (**4.6A, 4.7A, 4.7B**). Because the TEKS standard governing the illustration of the measure of an angle (**4.7A**) is not a tested standard, both understandings of angle measure are included in this module.



Topic C begins by decomposing 360° using pattern blocks, allowing students to see that a group of angles meeting at a point with no spaces or overlaps add up to 360° . With this new understanding, students now discover that the combined measure of two adjacent angles on a line is 180° , that the combined measure of two adjacent angles meeting to form a right angle is 90° , and that vertically opposite angles have the same measure. These properties are then used to solve unknown angle problems (4.7E). The terms “supplementary angle” and “complementary angle” are included in order to expose students to this vocabulary but are not assessed.

An introduction to symmetry opens Topic D as students recognize lines of symmetry for two-dimensional figures, identify line-symmetric figures, and draw lines of symmetry (4.6B). Given one half of a line-symmetric figure and the line of symmetry, students draw the other half of the figure. This leads to their work with triangles. Students are introduced to the precise definition of a triangle and then classify triangles based on angle measure (4.6D). Students construct triangles given a set of classifying criteria (e.g., create a triangle that is both right and isosceles). Finally, students explore the definitions of familiar quadrilaterals and classify them based on their attributes, including angle measure and parallel and perpendicular lines (4.6D). This work builds on Grade 3 reasoning about the attributes of shapes and lays a foundation for hierarchical classification of two-dimensional figures in Grade 5. The topic concludes as students compare and analyze two-dimensional figures according to their properties and use grid paper to construct two-dimensional figures given a set of criteria.

The Mid-Module Assessment follows Topic B. The End-of-Module Assessment follows Topic D.

Notes on Pacing for Differentiation

Module 4 may be taught after Module 6 and lessons truncated using the Preparing a Lesson protocol (see Module 1 Overview). This would change the order of the modules to the following: Modules 1, 2, 3, 5, 6, 4, and 7.

Alternately, Module 4’s lessons may be truncated by teaching Topic A simultaneously with Module 3 during an art class.

Topics B and C could be taught directly following Module 3, prior to Module 5, since they offer excellent scaffolding for the fraction work of Module 5. Topic D could be taught simultaneously with Module 5, 6, or 7 during an art class when students are served well with hands-on, rigorous experiences.

Keep in mind that Topics B and C of this module are foundational to Grade 7’s missing angle problems.

Focus Grade Level Standards

Geometry and Measurement

The student applies mathematical process standards to analyze geometric attributes in order to develop generalizations about their properties. The student is expected to:

- 4.6A** identify points, lines, line segments, rays, angles, and perpendicular and parallel lines;
- 4.6B** identify and draw one or more lines of symmetry, if they exist, for a two-dimensional figure;
- 4.6C** apply knowledge of right angles to identify acute, right, and obtuse triangles;
- 4.6D** classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size.

Geometry and Measurement

The student applies mathematical process standards to solve problems involving angles less than or equal to 180 degrees. The student is expected to:

- 4.7A** illustrate the measure of an angle as the part of a circle whose center is at the vertex of the angle that is "cut out" by the rays of the angle. Angle measures are limited to whole numbers;
- 4.7B** illustrate degrees as the units used to measure an angle, where $\frac{1}{360}$ of any circle is one degree and an angle that "cuts" $\frac{n}{360}$ out of any circle whose center is at the angle's vertex has a measure of n degrees. Angle measures are limited to whole numbers;
- 4.7C** determine the approximate measures of angles in degrees to the nearest whole number using a protractor;
- 4.7D** draw an angle with a given measure;
- 4.7E** determine the measure of an unknown angle formed by two non-overlapping adjacent angles given one or both angle measures.

Foundational Standards

Number and Operations

The student applies mathematical process standards to develop and use strategies and methods for whole number computations in order to solve problems with efficiency and accuracy. The student is expected to:

- 3.4K** solve one-step and two-step problems involving multiplication and division within 100 using strategies based on objects; pictorial models, including arrays, area models, and equal groups; properties of operations; or recall of facts.

Algebraic Reasoning

The student applies mathematical process standards to analyze and create patterns and relationships. The student is expected to:

- 3.5A** represent one- and two-step problems involving addition and subtraction of whole numbers to 1,000 using pictorial models, number lines, and equations;
- 3.5B** represent and solve one- and two-step multiplication and division problems within 100 using arrays, strip diagrams, and equations.

Focus Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

- MPS(C)** select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
- MPS(E)** create and use representations to organize, record, and communicate mathematical ideas;
- MPS(F)** analyze mathematical relationships to connect and communicate mathematical ideas;
- MPS(G)** display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Overview of Module Topics and Lesson Objectives

TEKS	ELPS	Topics and Objectives	Days
4.6A 4.6C	1.C 1.F 2.C 2.E 2.I 3.D 3.F 4.F 5.G	A Lines and Angles Lesson 1: Identify and draw points, lines, line segments, rays, and angles. Recognize them in various contexts and familiar figures. Lesson 2: Use right angles to determine whether angles are equal to, greater than, or less than right angles. Draw right, obtuse, and acute angles. Lesson 3: Identify, define, and draw perpendicular lines. Lesson 4: Identify, define, and draw parallel lines.	4
4.7A 4.7B 4.7C 4.7D	1.C 1.E 2.C 2.E 2.G 2.I 3.E 4.F 5.G	B Angle Measurement Lesson 5: Use a circular protractor to understand a 1-degree angle as $\frac{1}{360}$ of any circle. Explore benchmark angles using the protractor. Lesson 6: Use varied protractors to distinguish angle measure from length measurement. Lesson 7: Measure and draw angles. Sketch given angle measures, and verify with a protractor. Lesson 8: Identify and measure angles as turns and recognize them in various contexts.	4
		Mid-Module Assessment: Topics A–B (assessment $\frac{1}{2}$ day, return $\frac{1}{2}$ day, remediation or further application 1 day)	2
4.7E	1.C 2.E 2.H 2.I 3.E 3.H 4.D	C Problem Solving with the Addition of Angle Measures Lesson 9: Decompose angles using pattern blocks. Lessons 10–11: Use the addition of adjacent angle measures to solve problems using a letter for the unknown angle measure.	3

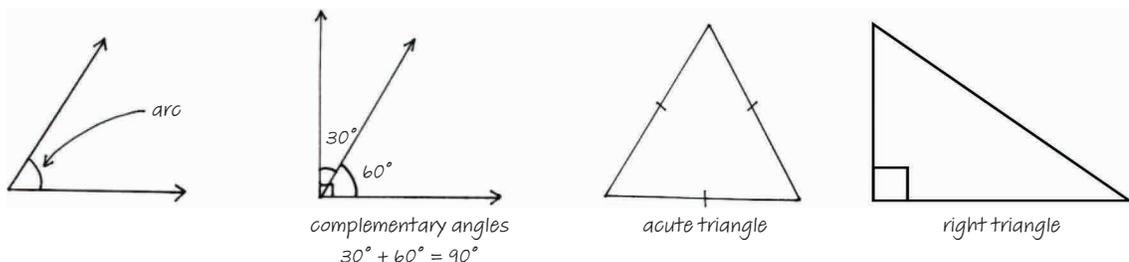


TEKS	ELPS	Topics and Objectives	Days
4.6A 4.6B 4.6C 4.6D	1.C 1.E 1.H 2.C 2.E 2.I 3.E 4.F 5.G	D Two-Dimensional Figures and Symmetry Lesson 12: Recognize lines of symmetry for given two-dimensional figures. Identify line-symmetric figures, and draw lines of symmetry. Lesson 13: Analyze and classify triangles based on angle measure. Lesson 14: Define and construct triangles from given criteria. Lesson 15: Classify quadrilaterals based on parallel and perpendicular lines and the presence or absence of angles of a specified size. Lesson 16: Reason about attributes to construct quadrilaterals on square or triangular grid paper.	5
		End-of-Module Assessment: Topics A–D (assessment ½ day, return ½ day, remediation or further application 1 day)	2
Total Number of Instructional Days			20

Terminology

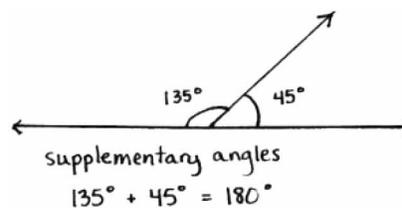
New or Recently Introduced Terms

- Acute angle (angle with a measure of less than 90°)
- Acute triangle (triangle with all interior angles measuring less than 90°)
- Adjacent angle (Two angles $\angle AOC$ and $\angle COB$, with a common side \overline{OC} , are *adjacent angles* if C is in the interior of $\angle AOB$.)
- Angle (union of two different rays sharing a common vertex, e.g., $\angle ABC$)
- Arc (connected portion of a circle)



- Collinear (Three or more points are *collinear* if there is a line containing all of the points; otherwise, the points are *non-collinear*.)

- Complementary angles (two angles with a sum of 90°)
- Degree, degree measure of an angle (Subdivide the length around a circle into 360 arcs of equal length. A central angle for any of these arcs is called a *one-degree angle* and is said to have an angle measure of 1° .)
- Diagonal (straight lines joining two opposite corners of a straight-sided shape)
- Figure (set of points in the plane)
- Interior of an angle (the convex¹ region defined by the angle)
- Intersecting lines (lines that contain at least one point in common)
- Length of an arc (circular distance around the arc)
- Line (straight path with no thickness that extends in both directions without end, e.g., \overleftrightarrow{AB})
- Line of symmetry (line through a figure such that when the figure is folded along the line, two halves are created that match up exactly)
- Line segment (two points, A and B , together with the set of points on \overleftrightarrow{AB} between A and B , e.g., \overline{AB})
- Obtuse angle (angle with a measure greater than 90° , but less than 180°)
- Obtuse triangle (triangle with an interior obtuse angle)
- Parallel (two lines in a plane that do not intersect, e.g., $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$)
- Perpendicular (Two lines are *perpendicular* if they intersect, and any of the angles formed between the lines is a 90° angle, e.g., $\overleftrightarrow{EF} \perp \overleftrightarrow{GH}$.)
- Point (precise location in the plane)
- Protractor (instrument used in measuring or sketching angles)
- Ray (The O is the point O and the set of all points on \overleftrightarrow{OA} that are on the same side of O as the point A .)
- Right angle (angle formed by perpendicular lines, measuring 90°)
- Right triangle (triangle that contains one 90° angle)
- Straight angle (angle that measures 180°)
- Supplementary angles (two angles with a sum of 180°)
- Triangle (A *triangle* consists of three non-collinear points and the three line segments between them. The three segments are called the *sides* of the triangle, and the three points are called the *vertices*.)
- Vertex (a point, often used to refer to the point where two lines meet, such as in an angle or the corner of a triangle)
- Vertical angles (When two lines intersect, any two non-adjacent angles formed by those lines are called *vertical angles* or *vertically opposite angles*.)



¹In Grade 4, a picture will suffice. A precise definition of convexity is given in high school geometry.

Familiar Terms and Symbols

- Decompose (process of separating something into smaller components)
- Parallelogram (quadrilateral with two pairs of parallel sides)
- Polygon (closed two-dimensional figure with straight sides)
- Quadrilateral (polygon with four sides)
- Rectangle (quadrilateral with four right angles)
- Rhombus (quadrilateral with all sides of equal length)
- Square (rectangle with all sides of equal length)
- Sum (result of adding two or more numbers)
- Trapezoid (quadrilateral with at least one pair of parallel sides)

Suggested Tools and Representations

- Folded paper models
- Pattern blocks
- Protractors of various diameters, including a 360° and 180° protractor
- Rectangular and triangular grid paper
- Right angle template (created in Lesson 2), set square
- Ruler (used to measure length), straightedge (used to draw straight lines)

Scaffolds

The scaffolds integrated into *A Story of Units*® give alternatives for how students access information as well as express and demonstrate their learning. Strategically placed margin notes are provided within each lesson elaborating on the use of specific scaffolds at applicable times. They address many needs presented by English language learners, students with disabilities, students performing above grade level, and students performing below grade level. Many of the suggestions are organized by Universal Design for Learning (UDL) principles and are applicable to more than one population.

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	4.6A 4.7A–D
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	4.6A–D 4.7A–E