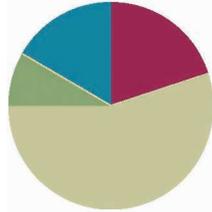


Lesson 24

Objective: Divide two- and three-digit dividends by two-digit divisors with single-digit quotients, and make connections to a written method.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Application Problem	(5 minutes)
■ Concept Development	(33 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Group Count by Multi-Digit Numbers **4.4D** (3 minutes)
- Estimate and Divide **5.3A, 5.3C** (4 minutes)
- Divide by Multiples of Ten with Remainders **5.3C** (5 minutes)

Group Count by Multi-Digit Numbers (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity prepares students for this lesson’s Concept Development.

Direct the students to count by 5–10 multiples of 21 forward and backward, occasionally changing directions and attempting to avoid student frustration.

Repeat process for 43.

Estimate and Divide (4 minutes)

Materials: (S) Personal white board

Note: This fluency exercise reviews Lesson 21 content.

Repeat the process from Lesson 22 for the following possible sequence: $607 \div 19$, $123 \div 24$, $891 \div 96$, and $5,482 \div 62$.

Divide by Multiples of Ten with Remainders (5 minutes)

Materials: (S) Personal white board

Note: This exercise reviews Lesson 19 content.

T: (Write $73 \div 50$.) On your personal white boards, solve the division problem using the standard algorithm. Check your work using multiplication and addition.

Repeat process for $70 \div 30$, $157 \div 30$, and $432 \div 70$.

Application Problem (5 minutes)

Billy has 2.4 m of ribbon for crafts. He wants to share it evenly with 12 friends. How many centimeters of ribbon would 7 friends get?

$$\begin{array}{l}
 2.4 \text{ m} = \underline{\quad} \text{ cm} \\
 2.4 \times 1 \text{ m} = \\
 2.4 \times 100 \text{ cm} = \\
 240 \text{ cm} \\
 240 \div 12 = \\
 (24 \times 10) \div 12 = \\
 (24 \div 12) \times 10 = \\
 2 \times 10 = 20
 \end{array}
 \qquad
 \begin{array}{l}
 20 \times 7 = \\
 (2 \times 10) \times 7 = \\
 (2 \times 7) \times 10 = \\
 14 \times 10 = \\
 140
 \end{array}$$

Seven friends will get 140 cm of ribbon.

Note: This Application Problem reaches back to concepts taught in Grade 5 Module 1.



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Students have a choice of strategies they can use to solve this Application Problem. They can think of 2.4 as 24 tenths. 24 tenths divided by 12 is 2 tenths. They can also compensate. Students can multiply the whole, 2.4, by 10. After they divide 24 by 12, students need to divide the quotient by 10. Both of these methods were explored in Module 1.

Concept Development (33 minutes)

Materials: (S) Personal white board

Problem 1: $72 \div 21$

T: (Write $72 \div 21$ horizontally on the board.) What is our whole?

S: 72.

T: Find a multiple of 20 close to 72 that makes this division easy. Show me how to estimate the quotient on your personal white board.

S: (Show $60 \div 20 = 6 \div 2 = 3$.)

T: I see you chose 60. Why not choose 80 and estimate the quotient as 4?

S: Because 4×20 is 80, and that's already too big.

$$\begin{array}{l}
 72 \div 21 \\
 \text{Estimate} \quad \text{Solve} \quad \text{Check} \\
 \approx 60 \div 20 \\
 = 6 \div 2 \\
 = 3
 \end{array}
 \qquad
 \begin{array}{r}
 3 \\
 21 \overline{) 72} \\
 \underline{-63} \\
 9
 \end{array}
 \qquad
 \begin{array}{l}
 21 \times 3 = 63 \\
 63 + 9 = 72
 \end{array}$$

Yes. The estimated quotient of 3 is perfect.

- T: Right, so our estimate means that there are about 3 twenty-ones in 72. Let's record that estimate. Where should it be recorded? (Write and set up the standard algorithm below $72 \div 21$ on the board.)
- S: In the ones place.
- T: What is 3×21 ?
- S: 63.
- T: (Record 63 below 72.) So, we've distributed 3 units of 21. How many of the 72 remain? Give me the full subtraction sentence.
- S: $72 - 63 = 9$.
- T: Is 9 enough to make another group of 21?
- S: No.
- T: How did our estimate help us solve the problem? Turn and share with your partner.
- S: We divided 60 by 20 to get our estimate, which was 3 ones. So, that's what we tried first in the quotient. → Our estimated quotient was 3, and it turned out that our actual quotient was 3 with a leftover of 9.
- T: Great. Let's check our answer. Whisper the number sentences to your partner.
- T: If I have 3 groups of 21 and add 9, what should my total be?
- S: 72.
- T: If I have 21 groups with 3 in each and 9 more, what should my total be?
- S: 72. → It's the same thing: 21 groups of 3 and 3 groups of 21 are both just 3×21 .
- T: Then, that means that when using the algorithm, we can view the divisor as either the number of groups or the size of each group.

Problem 2: $94 \div 43$

- T: (Write $94 \div 43$ horizontally on the board.) Use your personal white boards. Work with a partner.
1. Round the divisor.
 2. Find a multiple of the divisor that makes the division easy.
 3. Estimate the quotient.
 4. Solve using the standard algorithm.

T: Partner A will divide using the standard algorithm, and Partner B will check the answer. (Allow time for students to work.)

T: Partner A, say the quotient and the remainder for $94 \div 43$.

S: The quotient is 2, and the remainder is 8.

T: What does the quotient, 2, represent?

S: 2 groups of 43. → 43 groups with 2 in each one.

T: What does the remainder of 8 represent?

S: After 43 groups were made, 8 were left over. → We have 8 for the next group. → 8 that couldn't be distributed fairly into 43 groups.

T: Partner B, say your number sentences for checking the problem.

S: $43 \times 2 = 86$, and $86 + 8 = 94$.

Handwritten student work for $94 \div 43$:

Estimate: $\approx 80 \div 40 = 8 \div 4 = 2$

Solve:
$$\begin{array}{r} 2 \\ 43 \overline{) 94} \\ \underline{- 86} \\ 8 \end{array}$$

Check: $43 \times 2 = 86$
 $86 + 8 = 94$

A stick figure says: "Yes. The estimated quotient of 2 is perfect."

T: Again, let's look at our estimated quotient and our actual quotient. Did our estimated quotient turn out to be the actual quotient?

S: Yes.

Problem 3: $84 \div 23$

T: (Write $84 \div 23$ horizontally on the board.) We need a multiple of 20 that will make this division easy. Show me how to estimate the quotient.

S: $80 \div 20 = 8 \div 2 = 4$.

T: What are other ways of estimating this problem?

S: $90 \div 30 = 9 \div 3 = 3$. $\rightarrow 100 \div 25 = 4$.

T: These are all good ideas. Let's use our first possibility. (Write $80 \div 20 = 4$ on the board.) Let's now solve this problem using the standard algorithm. (Write and set up the standard algorithm below $84 \div 23$ on the board.) Our estimated quotient was 4, so I'll put 4 as the quotient. (Record 4 as the quotient in the ones place in standard algorithm.)

T: What are 4 units of 23?

S: 92.

T: Wait a minute! Let's stop and think. We have 84 in our total. Do we have enough to make 4 units of 23?

S: No.

T: What's happening here? Why didn't our estimated quotient work this time? Turn and discuss with your partner.

S: Our estimation sentence was correct. $84 \div 23$ becomes $80 \div 20 = 4$. \rightarrow We rounded our divisor down from 23 to 20. When we multiply 23 times 4, the product is 92. The product of 20 times 4 is 80. The extra part came from 4×3 . \rightarrow I know. We made the divisor smaller. The real divisor was bigger, so that means we are going to make fewer units. \rightarrow Yeah! If the divisor was just two more, 25, we would have rounded to 30, and then 90 divided by 30 is obviously 3.

T: So, if 4 ones is too big to be the quotient, what should we do?

S: Let's try 3.

T: How much is 3×23 ?

S: 69.

T: Take away those that we've distributed.

T: How many ones are remaining?

S: 15.

$84 \div 23$

estimate solve check

$\approx 80 \div 20$
 $= 8 \div 2$
 $= 4$

$23 \overline{) 84}$
 $\underline{-92}$

\Rightarrow

$23 \overline{) 84}$
 $\underline{-69}$
 15

$23 \times 3 = 69$
 $69 + 15 = 84$

No. The estimated quotient of 4 is too big. Try 3.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

For Problem 3, some students may easily see that 4 is an overestimate. Encourage these students to solve the problem by simply skip-counting by 23. They could also round the whole and keep the divisor unchanged while they estimate. This cultivates their number sense and challenges them appropriately. Students could also be asked, "What is the largest the whole could be and still have a quotient of 3?"

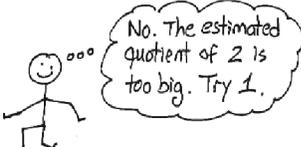
- T: What does the remainder of 15 tell us?
- S: We don't have enough for a fourth group. Those 15 ones are left over. → We'll need 8 more to make another group of 23.
- T: Give me the quotient and remainder for $84 \div 23$.
- S: The quotient is 3, and the remainder is 15.
- T: Whisper to your partner what these numbers represent and how we should check this problem.
- S: The 3 is 3 groups of 23, and the 15 are the ones that weren't enough to make another group. → We should multiply the quotient and the divisor, and then add the remainder.
- T: Say the multiplication sentence starting with 23.
- S: $23 \times 3 = 69$.
- T: (Record $23 \times 3 = 69$ horizontally on the board.) Say the addition sentence starting with 69.
- S: $69 + 15 = 84$.
- T: (Record $69 + 15 = 84$ below $23 \times 3 = 69$ on the board.) Is 84 our original whole?
- S: Yes, we solved it correctly.
- T: What did we just learn about estimated quotients? Turn and discuss.
- S: We should always estimate before we solve, but we may need to adjust it. → If we change the divisor or the whole a lot, it could make our estimate too big or too small.

Problem 4: $57 \div 29$

- T: (Write $57 \div 29$ horizontally on the board.) Use your personal white board. Work on this problem independently. Remember to estimate, divide, and check. Compare your work with a partner when you're finished.
- T: Tell me how you estimated.
- S: $60 \div 30 = 6 \div 3 = 2$.
- T: Can I use the quotient of 2? Discuss with your neighbor.
- S: No.
- T: Why not? How much is 2 units of 29?
- S: 58. 58 is greater than our whole of 57.
- T: So, what's the actual quotient?
- S: 1.
- T: Give me the quotient and remainder for $57 \div 29$.
- S: The quotient is 1 with a remainder of 28.
- T: What are the sentences for checking the problem?
- S: $29 \times 1 = 29$ and $29 + 28 = 57$.
- T: Talk to your partner about how we could create another division problem whose quotient is also 1 and whose remainder is 28.
- S: Just put any number in place of 29 in the check sentences, and get a new whole. We could use $34 \times 1 + 28 = 62$. So, $62 \div 34$ is also 1 R28. → We need 1 group of a number, and then we would add 28 to that. That will give us a new whole.

$57 \div 29$

<p><i>estimate</i></p> $\approx 60 \div 30$ $= 6 \div 3$ $= 2$	<p><i>solve</i></p> $\begin{array}{r} 29 \overline{) 57} \\ \underline{-58} \\ 28 \end{array}$	<p>⇒</p>	<p><i>check</i></p> $\begin{array}{r} 29 \overline{) 57} \\ \underline{-29} \\ 28 \end{array}$	<p><i>check</i></p> $29 \times 1 = 29$ $29 + 28 = 57$
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Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Divide two- and three-digit dividends by two-digit divisors with single digit quotients, and make connections to a written method.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- What pattern did you notice between Problems 1(c) and 1(f)?
- Did your initial estimates work for every example in Problem (1)? Why or why not? What happened in 1(d)?
- In Problem 2, what would you tell Linda in order to help her solve the problem? What lesson does Linda need to learn? What is another way that Linda could have estimated that would have eliminated the issue she encountered in the standard algorithm?
- Explain your thought process as you set up and began to solve Problems 3 and 4. What was challenging or unique about them? (Generating a division problem with the same quotient and remainder appears on the End-of-Module Assessment. Make time to debrief the students' thinking about Problem 4 thoroughly.)
- Talk about the importance of estimation when dividing with two-digit divisors.

Name Fan Date _____

1. Divide. Then check with multiplication. The first one is done for you.

a. $65 \div 17$ Check: $17 \times 3 = 51$

$$\begin{array}{r} 3 \text{ R } 14 \\ 17 \overline{) 65} \\ \underline{- 51} \\ 14 \end{array}$$

$$\begin{array}{r} 21 \\ \times 2 \\ \hline 42 \\ + 140 \\ \hline 490 \end{array}$$

b. $49 \div 21$ Check: $21 \times 2 = 42$

$$\begin{array}{r} 2 \text{ R } 7 \\ 21 \overline{) 49} \\ \underline{- 42} \\ 7 \end{array}$$

$$\begin{array}{r} 21 \\ \times 2 \\ \hline 42 \\ + 140 \\ \hline 490 \end{array}$$

c. $78 \div 39$ Check: $39 \times 2 = 78$

$$\begin{array}{r} 2 \\ 39 \overline{) 78} \\ \underline{- 78} \\ 0 \end{array}$$

$$\begin{array}{r} 39 \\ \times 2 \\ \hline 78 \end{array}$$

d. $84 \div 32$ Check: $32 \times 2 = 64$

$$\begin{array}{r} 2 \text{ R } 20 \\ 32 \overline{) 84} \\ \underline{- 64} \\ 20 \end{array}$$

$$\begin{array}{r} 32 \\ \times 2 \\ \hline 64 \\ + 160 \\ \hline 200 \end{array}$$

e. $77 \div 25$ Check: $25 \times 3 = 75$

$$\begin{array}{r} 3 \text{ R } 2 \\ 25 \overline{) 77} \\ \underline{- 75} \\ 2 \end{array}$$

$$\begin{array}{r} 25 \\ \times 3 \\ \hline 75 \\ + 200 \\ \hline 770 \end{array}$$

f. $68 \div 17$ Check: $17 \times 4 = 68$

$$\begin{array}{r} 4 \\ 17 \overline{) 68} \\ \underline{- 68} \\ 0 \end{array}$$

$$\begin{array}{r} 17 \\ \times 4 \\ \hline 68 \end{array}$$

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

2. When dividing 82 by 43, Linda estimated the quotient to be 2. Examine Linda's work and explain what she needs to do next. On the right, show how you would solve the problem.

Linda's estimation:	Linda's work:	Your work:
$\begin{array}{r} 2 \\ 40 \overline{) 80} \end{array}$	$\begin{array}{r} 2 \\ 43 \overline{) 82} \\ \underline{- 86} \\ 77 \end{array}$	$\begin{array}{r} 1 \text{ R}39 \\ 43 \overline{) 82} \\ \underline{- 43} \\ 39 \end{array}$

Linda's estimation of $80 \div 40 = 2$ was fine. But when she divided, she realized that $43 \times 2 = 86$. She can't take away 86 from 82. It should be 1 group of 43. The quotient is 1 with a remainder of 39.

3. A number divided by 43 has a quotient of 3 with 28 as a remainder. Find the number. Show your work.

$$43 \overline{) ?} \text{ R}28 \rightarrow \begin{array}{r} 43 \\ \times 3 \\ \hline 129 \end{array} + \begin{array}{r} 28 \\ \hline 157 \end{array} \quad \text{The number was 157.}$$

4. Write another division problem that has a quotient of 3 and a remainder of 28.

$$? \overline{) ?} \text{ R}28 \rightarrow \begin{array}{r} 52 \\ \times 3 \\ \hline 156 \end{array} + \begin{array}{r} 28 \\ \hline 184 \end{array} \quad \text{Check: } \begin{array}{r} 3 \text{ R}28 \\ 52 \overline{) 184} \\ \underline{- 156} \\ 28 \end{array}$$

184 divided by 52 is equal to 3 with a remainder of 28.

5. Mrs. Silverstein sold 91 cupcakes at a food fair. The cupcakes were sold in boxes of "a baker's dozen," which is 13. She sold all the cupcakes at \$15 per box. How much money did she receive?

$$\begin{array}{r} 7 \\ 13 \overline{) 91} \\ \underline{- 91} \\ 0 \end{array} \quad \begin{array}{l} 1 \text{ unit} = \$15 \\ 7 \text{ units} = 7 \times \$15 \\ = \$105 \end{array} \quad \text{She received } \$105.$$

Name _____

Date _____

1. Divide. Then, check with multiplication. The first one is done for you.

a. $65 \div 17$

b. $49 \div 21$

$$\begin{array}{r} 3 \text{ R } 14 \\ 17 \overline{) 65} \\ \underline{- 51} \\ 14 \end{array}$$

Check:

$17 \times 3 = 51$

$51 + 14 = 65$

c. $78 \div 39$

d. $84 \div 32$

e. $77 \div 25$

f. $68 \div 17$

2. When dividing 82 by 43, Linda estimated the quotient to be 2. Examine Linda's work, and explain what she needs to do next. On the right, show how you would solve the problem.

Linda's Estimation:

$$40 \overline{) 80} \begin{array}{r} 2 \\ \end{array}$$

Linda's Work:

$$43 \overline{) 82} \begin{array}{r} 2 \\ - 86 \\ \hline ? ? \end{array}$$

Your Work:

$$43 \overline{) 82}$$

3. A number divided by 43 has a quotient of 3 with 28 as a remainder. Find the number. Show your work.



4. Write another division problem that has a quotient of 3 and a remainder of 28.
5. Mrs. Silverstein sold 91 cupcakes at a food fair. The cupcakes were sold in boxes of “a baker’s dozen,” which is 13. She sold all the cupcakes at \$15 per box. How much money did she receive?

Name _____

Date _____

Divide. Then, check with multiplication.

a. $78 \div 21$

b. $89 \div 37$

Name _____

Date _____

1. Divide. Then, check with multiplication. The first one is done for you.

a. $72 \div 31$

b. $89 \div 21$

$$\begin{array}{r} 2 \text{ R } 10 \\ 31 \overline{) 72} \\ - 62 \\ \hline 10 \end{array}$$

Check:

$31 \times 2 = 62$

$62 + 10 = 72$

c. $94 \div 33$

d. $67 \div 19$

e. $79 \div 25$

f. $83 \div 21$

2. A 91 square foot bathroom has a length of 13 feet. What is the width of the bathroom?
3. While preparing for a morning conference, Principal Corsetti is laying out 8 dozen bagels on square plates. Each plate can hold 14 bagels.
- How many plates of bagels will Mr. Corsetti have?
 - How many more bagels would be needed to fill the final plate with bagels?