

Grade 5 • Module 4

Multiplication and Division of Fractions

OVERVIEW

In Module 4, students learn to multiply fractions and begin working with fraction division.

Students begin Topic A by decomposing non-unit fractions and representing these decompositions as repeated addition sentences. They build on this concept by representing addition sentences as multiplication,

just as they did in Grade 4: $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 3 \times \frac{1}{4}$. They will apply this understanding of decomposition and

multiplication and use the associative property in order to multiply a whole number by a fraction. In using the associative property, student multiply and then consider the unit.

$$4 \times \frac{2}{3} = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = 4 \times 2 \text{ thirds} = (4 \times 2) \text{ thirds} = 8 \text{ thirds} = \frac{8}{3}$$

$$4 \times \frac{2}{3} = \frac{(4 \times 2)}{3} = \frac{8}{3}$$

$$n \times \frac{a}{b} = \frac{n \times a}{b}$$

Continuing in Topic A, students proceed to multiplying a whole number by a mixed unit number by applying the distributive property. They recognize that they are multiplying each part of a mixed number by the whole number, and notice how efficient it is to use the distributive property. The topic closes with solving multiplicative comparison word problems involving fractions, and interpretation of data presented in dot plots.

$$5 \times 3\frac{3}{4} = 5 \times \left(3 + \frac{3}{4}\right)$$

$$= (5 \times 3) + \left(5 \times \frac{3}{4}\right)$$

$$= 15 + \frac{15}{4}$$

$$= 15 + 3\frac{3}{4}$$

$$= 18\frac{3}{4}$$



In Topic B, students interpret finding a fraction of a set ($\frac{3}{4}$ of 24) as multiplication of a whole number by a fraction ($\frac{3}{4} \times 24$) and use strip diagrams to support their understandings (5.3I). This, in turn, leads students to see division by a whole number as being equivalent to multiplication by its reciprocal. That is, division by 2, for example, is the same as multiplication by $\frac{1}{2}$. Students also use the commutative property to relate a fraction of a set to the Topic A repeated addition interpretation of multiplication by a fraction. This offers opportunities for students to reason about various strategies for multiplying fractions and whole numbers. Students apply their knowledge of a fraction of a set and previous conversion experiences (with scaffolding from a conversion chart, if necessary) to find a fraction of a measurement, thus converting a larger unit to an equivalent smaller unit (e.g., $\frac{1}{3}$ minutes = 20 seconds and $2\frac{1}{4}$ feet = 27 inches).

Interpreting numerical expressions opens Topic C as students learn to evaluate expressions with parentheses, such as $3 \times (\frac{2}{3} - \frac{1}{5})$ or $\frac{2}{3} \times (7 + 9)$ (5.4F). They then learn to interpret numerical expressions, such as *3 times the difference between $\frac{2}{3}$ and $\frac{1}{5}$* or *two-thirds the sum of 7 and 9* (5.4E). Students generate word problems that lead to the same calculation (5.3I), such as “Kelly combined 7 ounces of carrot juice and 5 ounces of orange juice in a glass. Jack drank $\frac{2}{3}$ of the mixture. How much did Jack drink?” Solving word problems (5.3I) allows students to apply new knowledge of fraction multiplication in context, and strip diagrams are used to model multi-step problems requiring the use of addition, subtraction, and multiplication of fractions.

Topic D begins the work of division with both fractions and decimal fractions. Students use strip diagrams and number lines to reason about the division of a whole number by a unit fraction and a unit fraction by a whole number (5.3J, 5.3L). Using the same thinking developed in Module 2 to divide whole numbers, students reason about how many *fourths* are in 5 when considering such cases as $5 \div \frac{1}{4}$. They also reason about the size of the unit when $\frac{1}{4}$ is partitioned into 5 equal parts: $\frac{1}{4} \div 5$. Using this thinking as a backdrop, students are introduced to decimal fraction divisors and use equivalent fraction and place value thinking to reason about the size of quotients, calculate quotients, and sensibly place the decimal in quotients (5.3A, 5.3D, 5.3E, 5.3F, 5.3G, 5.3K).

In Topic E, students engage in activities designed to bring the Personal Financial Literacy standards for Grade 5 to life. Through the use of a consistent real-life scenario, students understand how to balance a simple budget. Then they differentiate between gross income and net income while defining income tax and payroll tax. This gives students an opportunity to use what they know about fraction of a set to do simple tax calculations (5.10A). Students understand the difference between sales tax and property tax and discuss the advantages and disadvantages of various forms of payment (5.10C, 5.10E).

The module concludes with Topic F, in which numerical expressions involving fraction-by-fraction multiplication are interpreted and evaluated (**5.4E, 5.4F**). Students create and solve word problems involving both multiplication and division of fractions and decimal fractions.

The Mid-Module Assessment is administered after Topic C, and the End-of-Module Assessment follows Topic F.

Notes on Pacing for Differentiation

Lessons 13 and 14 are both word problem lessons involving addition, subtraction, and multiplication with fractions. Omit Lesson 13, but include Problems 1 and 4 as part of Lesson 14. In Lesson 14, use Problems 4 and 5 as an extension or challenge for early finishers, and omit Problems 5 and 6 from the Homework.

Note: Looking ahead, Topic D of Module 5 includes drawing in 5 of the 6 geometry lessons. These drawings with the protractor are critical to the coherence of the geometry standards of Grade 4 and those of middle school. These drawings could be completed during Module 4 but at a different time of the day, such as art class or for morning work. It is best that drawing with the protractor be taught by the math teacher. This modification allows for the later consolidation of Lessons 16, 17, 18, and 19 in Module 5.

Focus Grade Level Standards

Number and Operations

The student applies mathematical process standards to develop and use strategies and methods for positive rational number computations in order to solve problems with efficiency and accuracy. The student is expected to:

- 5.3I** represent and solve multiplication of a whole number and a fraction that refers to the same whole using objects and pictorial models, including area models;
- 5.3J** represent division of a unit fraction by a whole number and the division of a whole number by a unit fraction such as $1/3 \div 7$ and $7 \div 1/3$ using objects and pictorial models, including area models;
- 5.3K** add and subtract positive rational numbers fluently;
- 5.3L** divide whole numbers by unit fractions and unit fractions by whole numbers.



Algebraic Reasoning

The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:

- 5.4E** describe the meaning of parentheses and brackets in a numeric expression;
- 5.4F** simplify numerical expressions that do not involve exponents, including up to two levels of grouping.

Geometry and Measurement

- 5.7** The student applies mathematical process standards to select appropriate units, strategies, and tools to solve problems involving measurement. The student is expected to solve problems by calculating conversions within a measurement system, customary or metric.

Data Analysis

The student applies mathematical process standards to solve problems by collecting, organizing, displaying, and interpreting data. The student is expected to:

- 5.9A** represent categorical data with bar graphs or frequency tables and numerical data, including data sets of measurements in fractions or decimals, with dot plots or stem-and-leaf plots

Personal Financial Literacy

The student applies mathematical process standards to manage one’s financial resources effectively for lifetime financial security. The student is expected to:

- 5.10A** define income tax, payroll tax, sales tax, and property tax;
- 5.10B** explain the difference between gross income and net income;
- 5.10C** identify the advantages and disadvantages of different methods of payment, including check, credit card, debit card, and electronic payments;
- 5.10D** develop a system for keeping and using financial records;
- 5.10E** describe actions that might be taken to balance a budget when expenses exceed income;
- 5.10F** balance a simple budget.

Foundational Standards

The student is expected to:

- 4.2E** represent decimals, including tenths and hundredths, using concrete and visual models and money;
- 4.2G** relate decimals to fractions that name tenths and hundredths;
- 4.2H** determine the corresponding decimal to the tenths or hundredths place of a specified point on a number line;
- 4.3A** represent a fraction a/b as a sum of fractions $1/b$, where a and b are whole numbers and $b > 0$, including when $a > b$;
- 4.3B** decompose a fraction in more than one way into a sum of fractions with the same denominator using concrete and pictorial models and recording results with symbolic representations;
- 4.3C** determine if two given fractions are equivalent using a variety of methods;
- 4.3D** compare two fractions with different numerators and different denominators and represent the comparison using the symbols $>$, $=$, or $<$;
- 4.3E** represent and solve addition and subtraction of fractions with equal denominators using objects and pictorial models that build to the number line and properties of operations;
- 4.3F** evaluate the reasonableness of sums and differences of fractions using benchmark fractions 0, $1/4$, $1/2$, $3/4$, and 1, referring to the same whole;
- 4.3G** represent fractions and decimals to the tenths or hundredths as distances from zero on a number line.
- 4.8B** convert measurements within the same measurement system, customary or metric, from a smaller unit into a larger unit or a larger unit into a smaller unit when given other equivalent measures represented in a table.

Focus Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

- MPS(C)** select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
- MPS(D)** communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
- MPS(F)** analyze mathematical relationships to connect and communicate mathematical ideas.

Overview of Module Topics and Lesson Objectives

TEKS	ELPS	Topics and Objectives	Days
5.3I 5.3K 5.9A	1.E 1.H 2.E 2.I 3.E 3.G 3.H 4.G 5.G	A Repeated Addition of Fractions as Multiplication Lesson 1: Decompose non-unit fractions and represent them as a whole number times a unit fraction using strip diagrams. Lessons 2–3: Represent the multiplication of n times a/b as $(n \times a)/b$ using the associative property and visual models. Lessons 4–5: Find the product of a whole number and a mixed number using the distributive property. Lesson 6: Solve multiplicative comparison word problems involving fractions. Lesson 7: Solve word problems involving the multiplication of a whole number and a fraction including those involving dot plots.	7
5.3I 5.7 5.3K	1.C 2.E 2.I 3.C 3.E 3.H 4.G 5.G	B Multiplication of a Whole Number by a Fraction Lesson 8: Find a fraction of a set concretely and pictorially. Lesson 9: Multiply any whole number by a fraction using strip diagrams. Lesson 10: Relate a fraction of a set to the repeated addition interpretation of fraction multiplication. Lesson 11: Find a fraction of a measurement, and solve word problems.	4



TEKS	ELPS	Topics and Objectives	Days
5.3I 5.4E 5.4F 5.7 5.3K	1.C 1.H 2.E 3.D 3.E 3.H 4.B 4.K 5.G	C Fraction Expressions and Word Problems Lesson 12: Compare and evaluate expressions with parentheses and brackets. Lessons 13–14: Solve and create fraction word problems involving addition, subtraction, and multiplication. Lesson 15: Convert measures involving whole numbers, and solve multi-step word problems. Lesson 16: Convert mixed unit measurements, and solve multi-step word problems.	5
		Mid-Module Assessment: Topics A–C (assessment $\frac{1}{2}$ day, return $\frac{1}{2}$ day, remediation or further applications 1 day)	2
5.3F 5.3G 5.3J 5.3L 5.3A 5.3D 5.3E 5.3K 5.4F	1.C 1.E 2.E 2.I 3.E 3.H 4.G 5.G	D Division of Fractions Lesson 17: Divide a whole number by a unit fraction. Lesson 18: Divide a unit fraction by a whole number. Lesson 19: Solve problems involving fraction division. Lesson 20: Write equations and word problems corresponding to strip and number line diagrams.	4
5.10A 5.10B 5.10C 5.10D 5.10E 5.10F	2.C 2.E 2.G 2.I 3.B 3.E 3.G 4.J 5.G	E Applying Fraction and Decimal Multiplication to Personal Financial Literacy Lesson 21: Balance a simple budget. Lesson 22: Explain the difference between gross income and net income. Define income tax and payroll tax. Lesson 23: Define sales tax and property tax. Lesson 24: Identify the advantages and disadvantages of different methods of payment.	4



TEKS	ELPS	Topics and Objectives	Days
5.4E 5.4F	2.E 2.I 3.E 4.K 5.G	F Interpretation of Numerical Expressions Lesson 25: Interpret and evaluate numerical expressions.	1
		End-of-Module Assessment: Topics A–F (assessment ½ day, return ½ day, remediation or further applications 2 days)	3
Total Number of Instructional Days			30

Terminology

New or Recently Introduced Terms

- Budget (a plan for saving and spending money)
- Balanced budget (a budget with expenses that are exactly equal to income)
- Expense (items we spend money on; cost or charge)
- Income (money received)
- Net income (the amount of money left after taxes and other expenses are subtracted)
- Gross income (total amount of money received before taxes and other expenses are subtracted)
- Income tax (money the government collects based on one’s income)
- Payroll tax (money that is collected to help fund some programs that help people when they retire)
- Property tax (an extra cost on something owned, such as a house or land)
- Retail cost (the price paid at a store to buy something)
- Wages (money that is paid or received for work or services)

Familiar Terms and Symbols¹

- Commutative property (e.g., $4 \times \frac{1}{2} = \frac{1}{2} \times 4$)
- Conversion factor
- Decimal fraction
- Denominator (denotes the fractional unit, e.g., fifths in 3 fifths, which is abbreviated to the 5 in $\frac{3}{5}$)
- Distribute (with reference to the distributive property, e.g., in $1\frac{2}{5} \times 15 = (1 \times 15) + (\frac{2}{5} \times 15)$)

¹ These are terms and symbols students have seen previously.

- Divide, division (partitioning a total into equal groups to show how many units in a whole, e.g., $5 \div \frac{1}{5} = 25$)
- Dot plot
- Equation (a statement that two expressions are equal, e.g., $3 \times 4 = 6 \times 2$)
- Equivalent fraction
- Expression
- Factors (numbers that are multiplied to obtain a product)
- Foot, mile, yard, inch, gallon, quart, pint, cup, pound, ounce, hour, minute, second
- Fraction greater than or equal to 1 (e.g., $\frac{7}{2}, 3\frac{1}{2}$, an abbreviation for $3 + \frac{1}{2}$)
- Fraction written in the largest possible unit (e.g., $\frac{3}{6} = \frac{1 \times 3}{2 \times 3} = \frac{1}{2}$ or 1 three out of 2 threes = $\frac{1}{2}$)
- Fractional unit (e.g., the fifth unit in 3 fifths denoted by the denominator 5 in $\frac{3}{5}$)
- Hundredth ($\frac{1}{100}$ or 0.01)
- Mixed number ($3\frac{1}{2}$, an abbreviation for $3 + \frac{1}{2}$)
- Numerator (denotes the count of fractional units, e.g., 3 in 3 fifths or 3 in $\frac{3}{5}$)
- Parentheses (symbols () used around a fact or numbers within an equation or expression)
- Quotient (the answer when one number is divided by another)
- Strip diagram (method for modeling problems)
- Tenth ($\frac{1}{10}$ or 0.1)
- Unit (one segment of a partitioned strip diagram)
- Unknown (the missing factor or quantity in multiplication or division)
- Whole unit (any unit partitioned into smaller, equally sized fractional units)

Suggested Tools and Representations

- Area models
- Number lines
- Strip diagrams

Scaffolds

The scaffolds integrated into *A Story of Units*® give alternatives for how students access information as well as express and demonstrate their learning. Strategically placed margin notes are provided within each lesson elaborating on the use of specific scaffolds at applicable times. They address many needs presented by English language learners, students with disabilities, students performing above grade level, and students performing below grade level. Many of the suggestions are organized by Universal Design for Learning (UDL) principles and are applicable to more than one population.

Assessment Summary

Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic C	Constructed response with rubric	5.3I 5.4E 5.4F 5.7 5.9A
End-of-Module Assessment Task	After Topic F	Constructed response with rubric	5.3I 5.3J 5.3K 5.3L 5.4E 5.4F 5.7 5.9A 5.10A 5.10B 5.10C 5.10D 5.10E 5.10F